Resonant Wave-Particle Manipulation Techniques: Alpha-Channeling and Negative Mass Effect

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2011

Resonant Wave-Particle Interaction

- Resonant interaction of an integrable system with a weak external perturbation:

$$H = H^0(\boldsymbol{J}) + \varepsilon \sum_{\boldsymbol{\ell}} H^1_{\boldsymbol{\ell}}(\boldsymbol{J}) \cos(\boldsymbol{\ell} \cdot \boldsymbol{\theta} - \omega t)$$

  Resonance condition: $\boldsymbol{\ell} \cdot \boldsymbol{\Omega}(\boldsymbol{J}) = \omega$

- Two interaction regimes:
  (A) No resonance overlap: **regular** system dynamics
  (B) Resonance overlap: **stochastic** system dynamics

- Resonant wave-particle interaction can lead to:
  - **Stochastic motion**:
    - Particle ejection
    - Particle heating and cooling
    - Redistribution of energy between particles
    - Phase-space engineering
  - **Regular motion**:
    - Phase-locking $\Rightarrow$ Negative mass effect
    - A new type of instability
    - Plasma wave manipulation

TOC: Alpha-Channeling in Mirror Machines

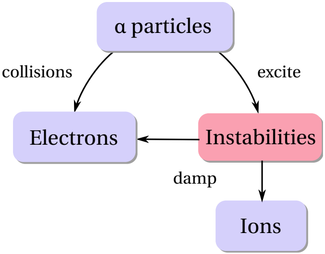
Alpha-Channeling Motivation

3.5 MeV alpha particles carry about 20% of the fusion energy

In mirror systems they:

1. Slow down on electrons
2. Take valuable electric potential
3. Excite energetic particle-driven modes
4. Susceptible to rapid radial losses

Energy flows



- R. Vann, H. Berk, and A. Soto-Chavez, *Phys. Rev. Lett.* **99**, 025003 (2007).
- J. Hanson and E. Ott, *Phys. Fluids* **27**, 51 (1984).
- N. Mizuno and M. Sato, *J. Phys. Soc. Jpn.* **51**, 1001 (1982).

Alpha-Channeling Motivation

Alpha channeling can:

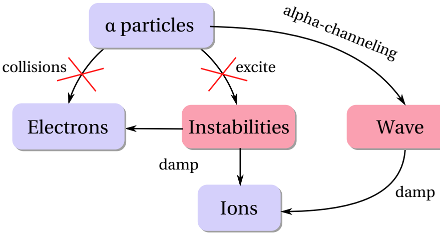
- Extract α particles from the device, redirecting energy to fuel ions
- Suppress energetic α particle-driven modes

What makes it possible?

α particles can be distinguished from fuel ions:

- very high energetic
- have a monoenergetic distribution
- localized in the device core

Energy flows



◦ N. J. Fisch and J. M. Rax, *Phys. Rev. Lett.* **69**, 612 (1992).

Quasilinear Diffusion

- Network of resonances $\boldsymbol{\ell} \cdot \boldsymbol{\Omega}(\boldsymbol{J}) = \omega$ in $\boldsymbol{J}$-space

$$H = H^0(\boldsymbol{J}) + \varepsilon \sum_{\boldsymbol{\ell}} H^1_{\boldsymbol{\ell}}(\boldsymbol{J}) \cos(\boldsymbol{\ell} \cdot \boldsymbol{\theta} - \omega t)$$

- Resonance overlap $\Rightarrow$ Stochastic dynamics
- Quasilinear diffusion

$$\frac{\partial f}{\partial t} = \sum_{i,j} \frac{\partial}{\partial J_i} \left(D^{\mathrm{QL}}_{ij} \frac{\partial f}{\partial J_j} \right),$$

  where

$$D^{\mathrm{QL}}_{ij} = \varepsilon^2 \sum_{\boldsymbol{\ell}} \ell_i \ell_j \pi \delta(\omega - \boldsymbol{\ell} \cdot \boldsymbol{J}) \left| H^1_{\boldsymbol{\ell}}(\boldsymbol{J}) \right|^2.$$

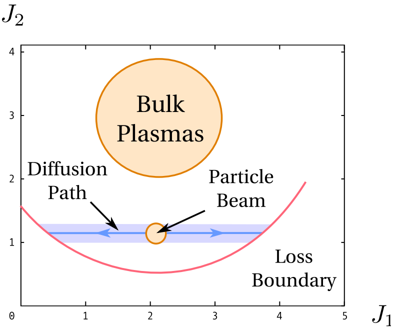
- The diffusion is locally directed along $\boldsymbol{\ell}$, i.e., $\Delta J_i / \Delta J_j = \ell_i / \ell_j$.

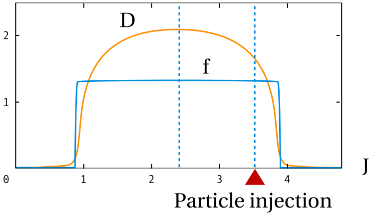
Diffusive Particle Ejection

- Wave spectrum ⇒ direction of diffusion and affected area

- In the presence of a loss boundary, the quasilinear diffusion can selectively eject particles

- No loss boundaries: Particle cooling or heating





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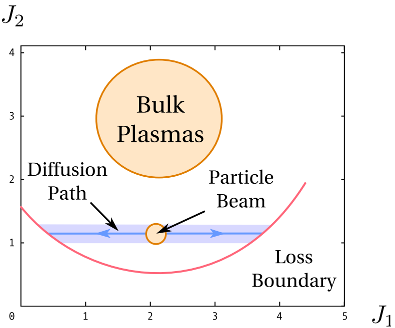
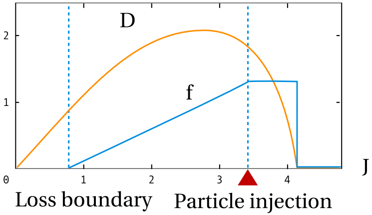
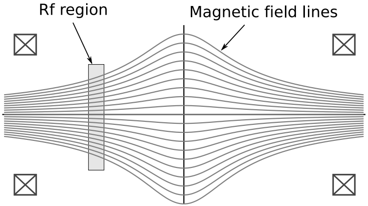


- Loss boundary: Particle ejection accompanied by particle cooling or heating



- Balance between the incoming and outgoing particle flows
- Energy difference ⇒ wave

Wave-Particle Interaction



- Resonance condition:

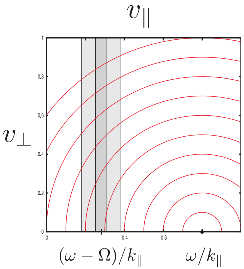
$$\omega - \ell\Omega - k_\| v_\| = 0$$

- Stochastic particle dynamics
- Anisotropic quasilinear diffusion

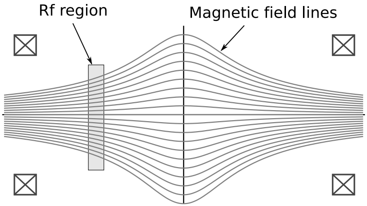
- Particle state kick:

$$\Delta p_\| = \frac{k_\| \Delta W_\perp}{\ell\Omega} \qquad \Delta R = -\frac{k_\perp \Delta W_\perp}{m\ell\Omega^2}$$

- Resonance condition requires $k_\| \Delta v_\| \to 0 \Rightarrow k_\| \to 0$ $(k_\| \ll \omega/v)$



Wave-Particle Interaction



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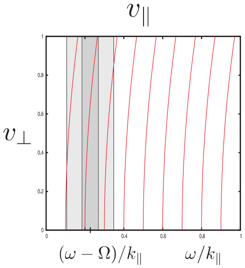
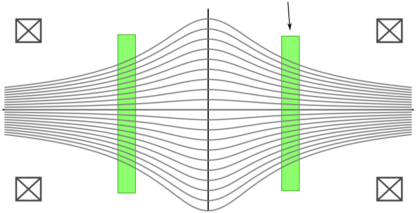
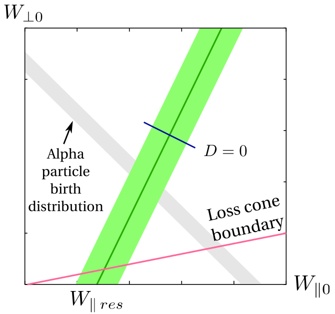
- Resonance condition requires $k_{\|}\Delta v_{\|} \to 0 \Rightarrow k_{\|} \to 0$ $(k_{\|} \ll \omega/v)$
- Random particle walk along the **diffusion path**



Device Design and System of Diffusion Paths

- Diffusion path shape: $\Delta p_{\|} \approx 0 \;\Rightarrow\; \boxed{W_{\|}^{0} \approx \left(\frac{\omega - \ell\Omega}{k_{\|}}\right)^{2} + W_{\perp}^{0}(B_{\text{wave}}/B_0 - 1)}$





- Limitation of diffusion paths:
  1. wave $k_{\perp}$
  2. wave radial profile $E_{\text{wave}}(R)$

- N. J. Fisch, *Phys. Rev. Lett.* **97**, 225001 (2006).
- A. I. Zhmoginov and N. J. Fisch, *Phys. Plasmas* **15**, 042506 (2008).

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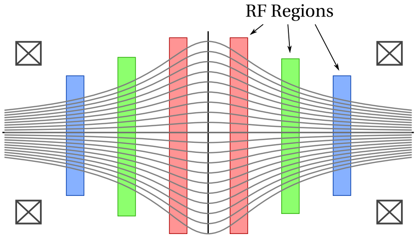

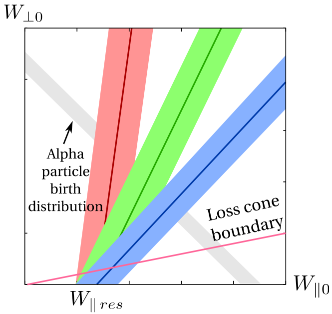

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Promote Diffusion in Favorable Direction

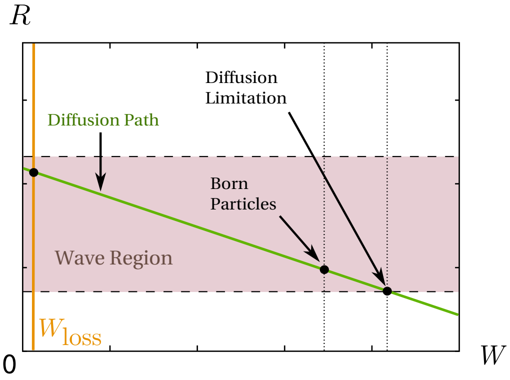
- For an electrostatic wave:

$$\boxed{D \sim E^2_{\mathrm{wave}}(R) J^2_\ell(k_\perp \rho)}$$

- Infinitely many zeroes: $J_\ell(z_i^{(\ell)}) = 0$
- $k_\perp \rho = z_i^{(\ell)} \;\Rightarrow\; D = 0$

- $E_{\mathrm{wave}}(R) = 0 \;\Rightarrow D = 0$
- Radial wave limitation $\Rightarrow \alpha$ heating limitation:

$$\boxed{\Delta R = -\frac{k_\perp \Delta W_\perp}{m\ell\Omega^2}}$$



Feasibility of Alpha-Channeling

To capture the full effect consider:

- $\vec{B}(r,z)$
- $E_{\mathrm{rf}}(r,z)$
- multiple overlapping diffusion paths

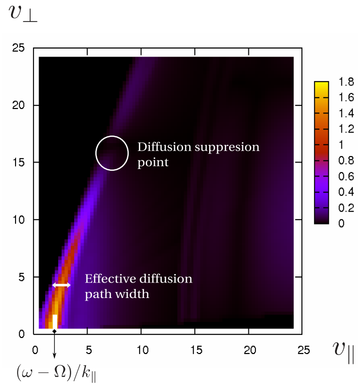
Methodology

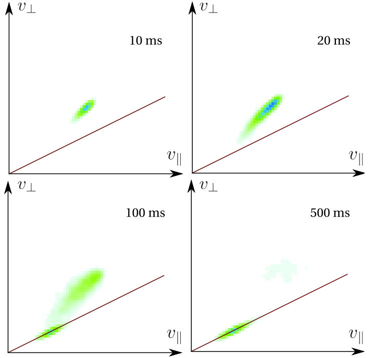
- Numerical simulations of:
  - Single particle motion
  - Particle diffusion by solving the Fokker-Planck equation
- Optimizing efficiency for α-particle birth distribution over wave parameters
- Perturbation and injection of fuel ions

- A. I. Zhmoginov and N. J. Fisch, *Phys. Plasmas* **15**, 042506 (2008).

Particle Diffusion Simulations

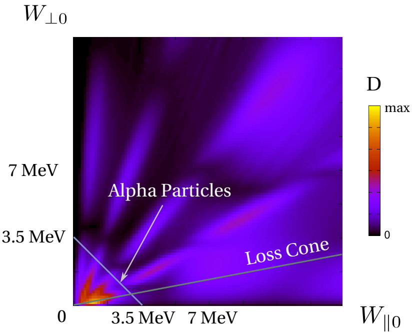
- Numerical examples showing: (a) *diffusion path* and (b) particle diffusion along a diffusion path:

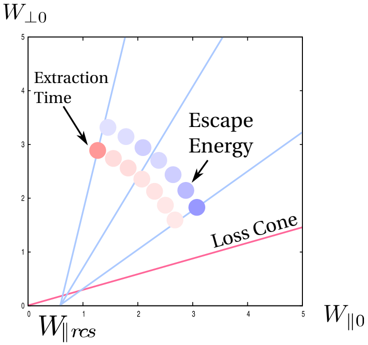




Particle Diffusion Simulation Results

- Typical device parameters: $L = 20\,\mathrm{m}$ to $40\,\mathrm{m}$, $B \sim 2\,\mathrm{T}$, $T \sim 20\,\mathrm{keV}$
- Results:
  1. 80% of α particles extracted
  2. 60% of α particle energy channeled (75% from extracted)
  3. deeply-trapped particles leave slowest, but with least energy





Main Results

Main Results

- Two codes solving particle motion equations and the Fokker-Planck equation are developed and compared
- The effect of the magnetic field inhomogeneity is discussed and diffusion path parameters are estimated
- The possibility to channel 60% of α particle energy (75% from extracted) is demonstrated
- The extraction is accomplished in 300 ms by 8 rf regions perturbing the background plasma density by less than 5%
- The perturbation of the background plasma is studied and shown to be sufficiently small for smooth wave profiles
- A possibility of injecting fuel ions using the α-channeling waves is demonstrated

What Waves are Suitable for Channeling

- Waves with optimal parameters should be identified in plasmas
- Weakly-damped localized modes require less power

Methodology

- New method for finding weakly-damped modes is proposed
- Code solving ray tracing equations is developed
- Identification of suitable modes in practical fusion devices is performed

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- A. I. Zhmoginov and N. J. Fisch, *Phys. Plasmas* **16**, 112511 (2009).
- A. I. Zhmoginov and N. J. Fisch, *Fus. Sci. Tech.* **57**, 361 (2010).

Wave and Device Parameters

- Requirements for the α-channeling mode:
  1. $\omega \approx n\Omega_\alpha$
  2. $k_\parallel \ll \omega/v$ and $k_\perp \rho_\alpha \geq 1$
  3. weakly damped, interacting with deeply-trapped α particles
  4. damping on ions at least comparable to the damping on electrons
- Considered device parameters:
  1. **Proof-of-Principle Facility**
     $d = 1.2\,\mathrm{m}$, $L = 12\,\mathrm{m}$, $T_{0\,e/i} = 4\,\mathrm{keV}$, $B \sim 1\,\mathrm{T}$, $n \sim 10^{13}\,\mathrm{cm}^{-3}$, $n_D/n_T = 1$
  2. **Fusion Reactor Prototype**
     $d = 6\,\mathrm{m}$, $L = 15\,\mathrm{m}$, $T_{0\,e} = 60\,\mathrm{keV}$, $T_{0\,i} = 15\,\mathrm{keV}$, $B \sim 3\,\mathrm{T}$, $n \sim 10^{14}\,\mathrm{cm}^{-3}$, $n_D/n_T = 1$
  3. **LAPD**
     $d = 60\,\mathrm{cm}$, $L = 10\,\mathrm{m}$, $T_{0\,e} = 5\,\mathrm{eV}$, $T_{0\,i} = 1\,\mathrm{eV}$, $B \sim 500\,\mathrm{G}$, $n \sim 2 \cdot 10^{12}\,\mathrm{cm}^{-3}$, $n_{He}/n_N = 4$

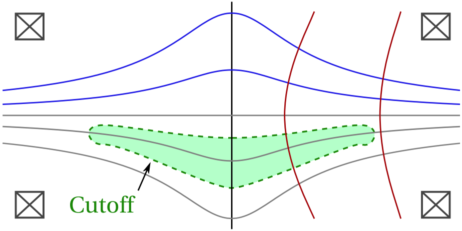
- J. Pratt, W. Horton, and H. L. Berk, *J. Fusion Energ.* **27**, 91 (2008).
- W. Gekelman, H. Pfister, Z. Lucky, J. Bamber, D. Leneman, and J. Maggs, *Rev. Sci. Instrum.* **62**, 2875 (1991).

Approach to the Wave Search

- WKB approximation:

$$\frac{d\boldsymbol{r}}{d\tau} = \frac{\partial \mathcal{D}}{\partial \boldsymbol{k}}, \quad \frac{d\boldsymbol{k}}{d\tau} = -\frac{\partial \mathcal{D}}{\partial \boldsymbol{r}}, \quad \frac{dt}{d\tau} = \frac{\partial \mathcal{D}}{\partial \omega}.$$

- Waves propagate along or across $\vec{B}$

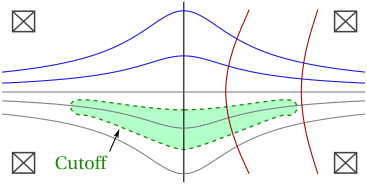


- A. I. Zhmoginov and N. J. Fisch, *Phys. Plasmas* **16**, 112511 (2009).
- A. I. Zhmoginov and N. J. Fisch, *Fus. Sci. Tech.* **57**, 361 (2010).

Curvilinear Coordinate System

- Fast and slow motions; adiabatic invariant coservation (fast motion)
- Introduce new coordinates $[R(r,z), \varkappa(r,z)]$:

$$\nabla R = \alpha(R,\varkappa)\left[\hat{b}_z, -\hat{b}_r\right], \qquad \nabla \varkappa = \beta(R,\varkappa)\left[\hat{b}_r, \hat{b}_z\right].$$



Metric tensor is diagonal:

$$\bar{g}_{ij} = \begin{pmatrix} \alpha^{-2} & 0 \\ 0 & \beta^{-2} \end{pmatrix}.$$

- Ray-tracing Hamiltonian in the new coordinates:

$$\mathcal{H}(K_R, K_\varkappa, R, \varkappa) = \mathcal{D}(\alpha K_R, \beta K_\varkappa, R, \varkappa)$$

where

$$\alpha K_R = k_r \hat{b}_z - k_z \hat{b}_r = k_n \qquad \beta K_\varkappa = k_r \hat{b}_r + k_z \hat{b}_z = k_\parallel$$

- Use full kinetic dispersion relation

Waves

- **Suitable waves**: fast and shear Alfvén waves, ion Bernstein wave
- **Dispersion relation of the identified waves**:

$$a = n_{\parallel}^2 + \frac{d^2}{b - n^2},$$

where

$$a \approx 1 - \sum_i \frac{\omega_{pi}^2}{\omega} \sum_n e^{-\lambda_i} \frac{n^2 I_n(\lambda_i)}{\lambda_i(\omega - n\Omega_i)}$$

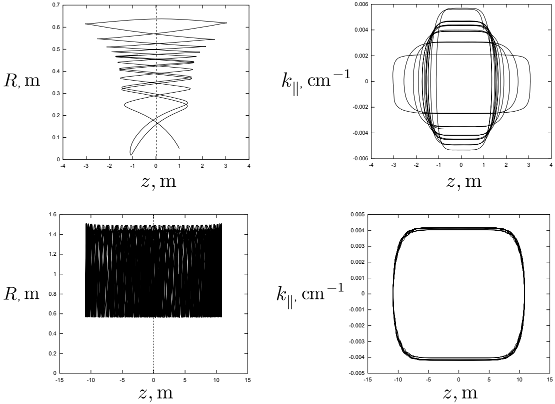
$$b \approx 1 - \sum_i \frac{\omega_{pi}^2}{\omega} \sum_n \frac{e^{-\lambda_i}}{\omega - n\Omega_i} \left[\frac{n^2 I_n}{\lambda_i} + 2\lambda_i \left(I_n - I_n'\right)\right]$$

$$d \approx \sum_i \frac{\omega_{pi}^2}{\omega} \sum_n \frac{n e^{-\lambda_i} \left(I_n - I_n'\right)}{\omega - n\Omega_i} + \frac{\omega_{pe}^2}{\omega\Omega_e}$$

- **Wave properties**:
  - $\omega \sim n\Omega_\alpha$, $v_{\mathrm{ph}} > v_{\mathrm{th}\,e}$, $\lambda_\parallel \sim 1\,\mathrm{m}$
  - $k_\perp \rho_i \ll 1$ for the cold waves ($\lambda_\perp \sim 10\,\mathrm{cm}$) and $k_\perp \rho_i \sim 1$ for kinetic wave ($\lambda_\perp \sim 1\,\mathrm{cm}$)

Waves

- **Suitable waves**: fast and shear Alfvén waves, ion Bernstein wave
- **Ray trajectories**:



- Sensitive to small magnetic field perturbations

Mode Structure Calculation

Approaches

- **WKB**: Large radial and longitudinal mode numbers
- **Integral Equation**: Small radial and longitudinal mode numbers
- **Semi-WKB**: Large radial, but not longitudinal mode numbers

- Original equation:
$$\hat{\mathcal{D}}_\omega \varphi(x,y) = 0$$
- Decomposition:
$$\varphi(x,y) = \sum_j \varphi_j(x,y) e^{iS_0(y)+iS_1(y)+\ldots}$$
- First two equations:
$$\hat{\mathcal{D}}_\omega^\circ \varphi_0 = 0$$
$$\hat{\mathcal{D}}_\omega^\circ \varphi_1 - \frac{iS_0''}{2}\frac{\partial^2 \hat{\mathcal{D}}_\omega^\circ}{\partial \hat{k}_y^2}\varphi_0 + S_1' \frac{\partial \hat{\mathcal{D}}_\omega^\circ}{\partial \hat{k}_y}\varphi_0 = 0$$

where $\hat{\mathcal{D}}_\omega^\circ = \hat{\mathcal{D}}_\omega(\hat{k}_x, S_0', x, y)$

Mode Structure Calculation

Approaches

- **WKB**: Large radial and longitudinal mode numbers
- **Integral Equation**: Small radial and longitudinal mode numbers
- **Semi-WKB**: Large radial, but not longitudinal mode numbers

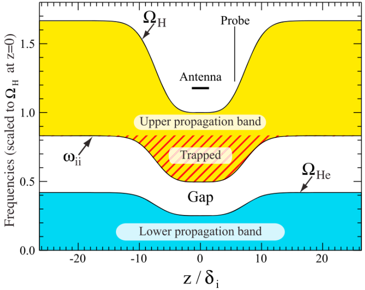
- Generalize to the integral operators of a special type
- For an electrostatic wave: Vlasov-Poisson equation in action-angle variables:

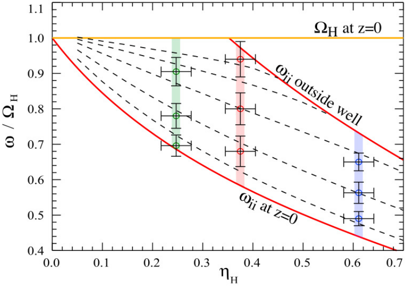
$$\Delta\varphi \approx 4\pi q_s^2 \int d^3 J \, \frac{\partial f_0}{\partial J_i} \int_0^\infty d\tau \, \frac{\partial J_i}{\partial p_j} \frac{\partial \varphi}{\partial q_j} \left| \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\theta}} \right|^{-1} .$$

- Using Monte-Carlo integration method:
  - may require less computational power
  - the error can be estimated
  - advanced integration methods use more samples where necessary
- Wave damping can be also calculated

Possible Experimental Verification

- The Large Plasma Device (LAPD) at UCLA could be used to verify the α-channeling concept and observe the identified α-channeling modes
- Presently, the fast Alfvén wave launching campaign is underway
- Shear Alfvén waves. Ion-ion hybrid Alfvén wave resonator observed





- S. Vincena, W. Farmer, J. Maggs, and G. Morales, *Geophys. Res. Lett.* **38**, L11101 (2011).
- M. Temerin and R. Lysak, *J. Geophys. Res.* **89**, 2849 (1984).
- V. Guglielmi, A. Potapov, and C. Russell, *JETP Lett.* **72**, 298 (2000).

Main Results

Main Results

- Quasi-one-dimensional wave propagation is studied
- A method of identifying weakly-damped localized modes is proposed
- Code calculating ray trajectories and identifying modes suitable for α-channeling is developed
- Modes suitable for α-channeling are identified in several practical device designs
- Crude dynamical model of α-channeling is analyzed
- Mode stability in the presence of periodic magnetic field fluctuations is addressed
- A method for calculating IBW mode structure is proposed
- Approaches to the experimental study of the identified modes are proposed

Minority Ion Catalysis

- Alpha-channeling modes damp on electrons
- Redirection of wave energy to fuel ions, requires:
  1. resonance with α particles
  2. resonance with fuel ions
- But this:
  1. restricts parameter space of waves
  2. need to avoid fuel pump-out

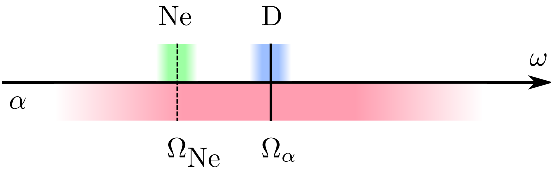
Possibilities

- Travelling waves: convective amplification followed by damping
- Contained modes:
  - mode grows on α particles and damps on ions
  - mode grows on α particles and damps on minorities (extends wave parameter space)

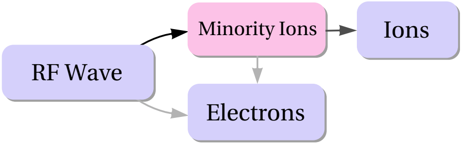
- E. J. Valeo and N. J. Fisch, *Phys. Rev. Lett.* **73**, 3536 (1994).
- A. I. Zhmoginov and N. J. Fisch, *Phys. Rev. Lett.* **107**, 175001 (2011).

Minority Ion Injection

- Wave energy redirected to fusion ions through minority ions
- Avoid plasma pump-out and strong wave damping $\Rightarrow \omega \ncong \Omega_{\mathrm{D}}$, $\omega \cong \Omega_{\min}$



- $\nu^{\min/\mathrm{D}} \gg \nu^{\min/\mathrm{e}}$.
- The minority ions heated by the wave can then dissipate their energy on ions through collisions

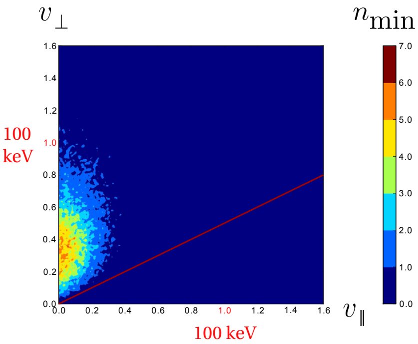


Minority Ion Injection

Conditions on the wave energy density:

- $T_i \ll T_{\text{min}} \ll 200\,\text{keV}$ ($\mathcal{W}_{\text{min}\to\text{ions}} > 0$ and $\mathcal{W}_{\text{min}\to\text{ions}} \gg \mathcal{W}_{\text{min}\to\text{electrons}}$)
- $\tau^{\alpha}_{\text{QL}} \ll \tau^{\alpha}_{\text{coll}}$
- $P_{\text{rf}} \ll P_{\text{crit}}$
- $\mathcal{W}_{\text{min loss}} \ll \mathcal{W}_{\text{wave}\to\text{ions}}$

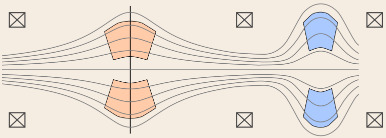
- Numerically solved Fokker-Planck equation for minorities and α particles
- Showed that all conditions can be satisfied for a practical device ($T \approx 10\,\text{keV}$, $n \approx 3 \cdot 10^{13}\,\text{cm}^{-3}$, $B \approx 1\,\text{T}$)
- Polarization of the fast wave is **crucial** for moderate minority cooling
- Damping independent of $k_{\parallel}$ and ω (in inhomogeneous magnetic field)

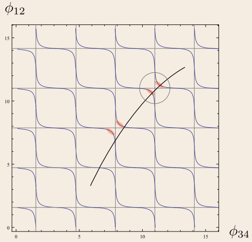


Main Results

Main Results

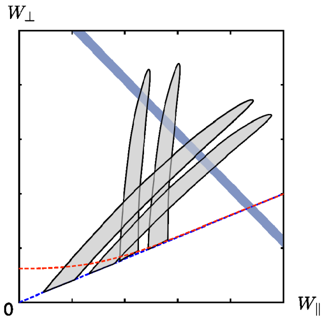
- Minority ion injection technique is proposed
- Code solving the collisional Fokker-Planck equations for minorities and α particles is developed
- The feasibility of the minority ion injection technique is demonstrated
- A method of coupling the localized mode in the central cell to the localized mode in the device plug is proposed

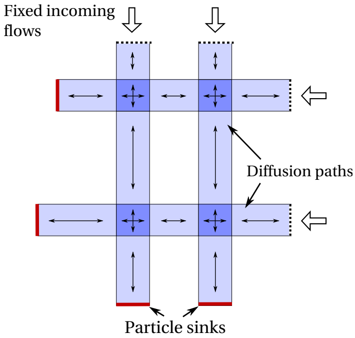




Origin of the Network of Diffusion Paths

- Quasilinear diffusion is a flexible tool for phase space manipulation
- “Network of diffusion paths”
- Manipulated by changing diffusion coefficients of individual paths

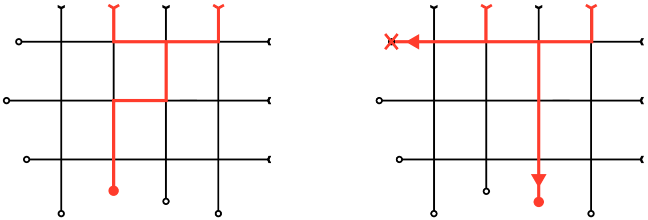




- A. Zhmoginov and N. Fisch, *Phys. Lett. A* 372, 5534 (2008).

Optimization Problem

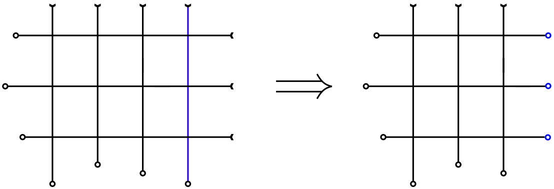
- Constrained flexibility:



- Minimization of the weighted sum S of outgoing fluxes given the input fluxes
- **Nonlinear** optimization problem:
  1. $S(\vec{D})$ is a nonlinear function; the optimization domain is $D_i > 0$
  2. $S(\vec{I})$ is a linear function; the optimization domain is complex
- Equivalence of the network to an electrical circuit:
  - Particle fluxes: currents
  - Particle densities: potentials
  - Loss boundary: grounded ends
  - Diffusion coefficients: conductivities

Solution of the Optimization Problem

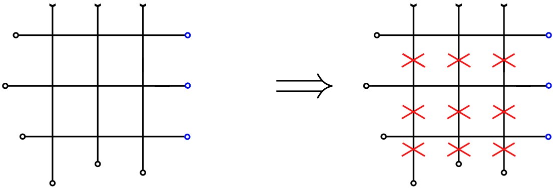
1. Excluding the minimum weighted path



2. **Constraints**:
   - $J = -D\frac{\Delta f}{\Delta x}$
   - $\sum\limits_{i} J_i = 0$

Solution of the Optimization Problem

3. Removing vertical constraints $J = -(\Delta f / \Delta y) D$



- $n + m - 1$ free parameters
- $\vec{D}$ or $\vec{I}$ are varied
- $S(\vec{D})$ is **nonlinear**
- $S(\vec{I})$ is linear, but **nonlinear** $\{\vec{I}\}$

- nm free parameters
- $\Delta J_{k\ell}$ are varied
- $nm + n$ **linear** inequalities
- $S(\Delta J_{k\ell})$ is **linear**

4. Considering some solution of the new **linear** optimization problem
5. Proving that there are many configurations with the same S
6. Proving that one of such configurations $\bar{f}_{ij}$ can be reached asymptotically in a network of diffusion paths

Main Results

Main results

- Numerical codes for solving two-dimensional diffusion equation and finding the optimum in a network of thin diffusion paths are developed
- It is proven that the minimum of S is reached in a network of diffusion paths with coefficients 1, β, β^2, β^3, β^4 as $\beta \to \infty$
- The manifold formed by possible $\{\vec{I}\}$ is shown to be non-convex
- The choice of diffusion coefficients is shown to be reduced to solving a linear optimization problem
- The derivative $\partial S / \partial D_{xk}$ is calculated explicitly

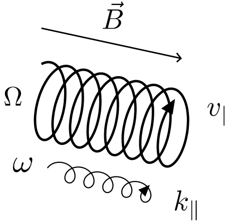
TOC: Negative Mass Effect

Motivation

Negative Mass Effect for the Vaccum Wave

I. Y. Dodin and N. J. Fisch, Phys. Rev. E **77**, 036402 (2008).

1. Systems undergoing slow oscillation-center drift and fast oscillatory motion interacting with external resonant force considered
2. Drift center Lagrangian and the *effective mass* introduced
3. Particle interaction with a **vacuum** circularly polarized wave in a background magnetic field studied
4. Tristability and negative parallel mass effect described



Perturbed Hamiltonian

- New formalism is necessary to study the non-vacuum ($n \neq 1$) case
- System Hamiltonian:

$$H = H_0(\boldsymbol{I}) + \varepsilon H_1(\boldsymbol{I}) \cos(\omega_0 t - \boldsymbol{\ell} \cdot \boldsymbol{\phi})$$

- Resonance condition: $\boldsymbol{\ell} \cdot \boldsymbol{\omega} = \omega_0$
- Canonical transformation

$$\theta_i = \phi_i \qquad \theta_n = \omega_0 t - \boldsymbol{\ell} \cdot \boldsymbol{\phi}$$
$$J_i = I_i - \ell_i I_n / \ell_n \qquad J_n = I_n / \ell_n$$

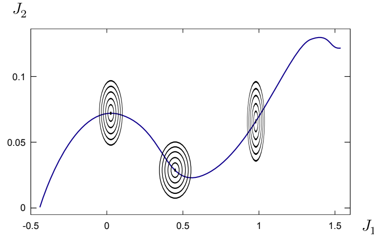
- New **1D** Hamiltonian

$$\boxed{\mathcal{H} = H_0 - \omega J_n + \varepsilon H_1 \cos \theta_n}$$

- Stationary points correspond to *externally-driven solutions* (with zero energy in internal degrees of freedom)

Stationary Points

- System Hamiltonian:
$$\mathcal{H} = H_0 - \omega J_n + \varepsilon H_1 \cos\theta_n$$
- $J_{i<n}$ are constants of motion
- Stationary points $\dot{J}_n = \dot{\theta}_n = 0$ form $(n-1)$-dimensional *stationary surfaces* in the n-dimensional $\boldsymbol{J}$-space:



- The particle is "locked" to the stationary surface: $J_{i<n}$ change slowly due to external forces $\Rightarrow$ adiabatic invariant $\Lambda = \oint J_n \, d\theta_n = \text{const}$
- Only dissipative forces can affect Λ, driving particles away or towards the resonance surface

Effective Mass

- Add external force F

$$\dot{I}_1 = \partial H/\partial\phi_1 + F \qquad \dot{I}_2 = \partial H/\partial\phi_2$$

- Then:

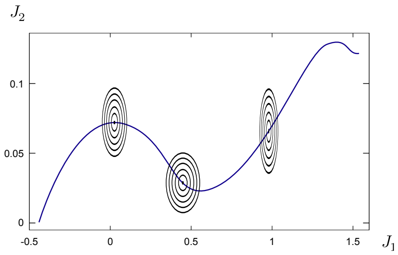
$$\dot{J}_1 = F \quad J_2 = J_2(J_1)$$
$$I_1 = J_1 + \ell_1 J_2$$

- Therefore:

$$\frac{d}{dt}\langle I_1\rangle = F\left(1 + \ell_1\frac{dJ_2}{dJ_1}\right)$$

- Effective mass:

$$\boxed{m_{\text{eff}} \equiv \left(1 + \ell_1\frac{dJ_2}{dJ_1}\right)^{-1}}$$



Wave-Particle Interaction

- Particle interacting with a circularly-polarized wave

$$\boldsymbol{A} = \frac{mc^2}{q}\frac{a_0}{\sqrt{2}}\left(\hat{\boldsymbol{x}}\cos\nu - \hat{\boldsymbol{y}}\sin\nu\right)$$

  and magnetic field $\boldsymbol{B} = \hat{\boldsymbol{z}}B_0$

- Particle Hamiltonian:

$$H \approx H_0 - \frac{\varepsilon\widetilde{\mu}^{1/2}}{H_0}\cos(\theta - \omega t + kz)$$

  where $H_0 = c(m^2c^2 + 2m\Omega_0\widetilde{\mu} + p^2)^{1/2}$, $\varepsilon = mc^3\sqrt{m\Omega_0}a_0$ and $\nu = \omega t - kz$

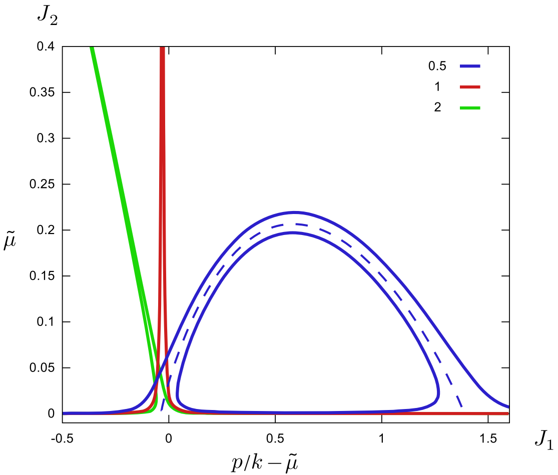
- Canonical momentum $\widetilde{\mu}$ is **not** equal to the magnetic momentum μ:

$$\mu = \left(\pm\sqrt{2m\Omega\widetilde{\mu}} - q|\boldsymbol{A}|/c\right)^2/(2m\Omega)$$

- Similar systems were studied previously for $n = 0$ and $n = 1$
- Here we consider the case $n \neq 1$

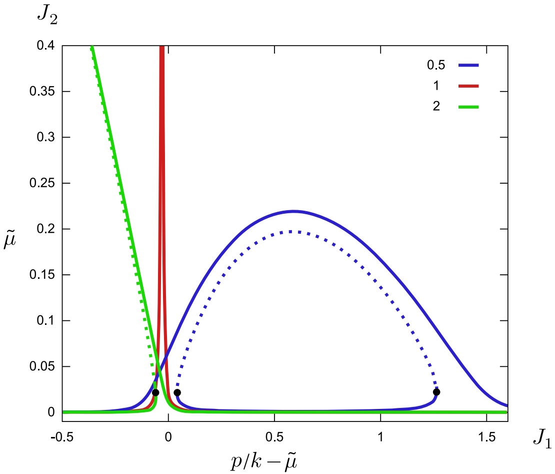
Stationary Curve

- Stationary curve in $(J_1, J_2) = (-\widetilde{\mu}, p/k - \widetilde{\mu})$ coordinates:



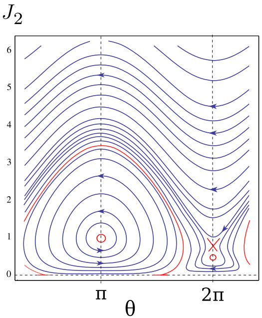
System Stability

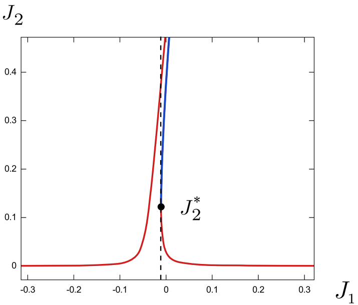
- Unstable regions of the stationary curve:



System Stability

- System stability determined using the Poincaré-Hopf index theorem





- The expression for the critical point obtained:

$$\boxed{J_2^* = \left(\frac{\varepsilon}{8|1-n|\omega^2}\right)^{2/3}}$$

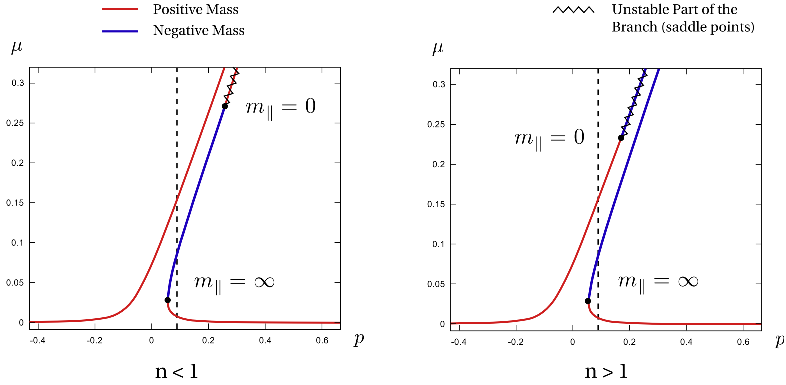
○ A. I. Zhmoginov, I. Y. Dodin and N. J. Fisch, *Phys. Rev. E* **81**, 036404 (2010).

Paralell Mass

- Parallel mass calculated:

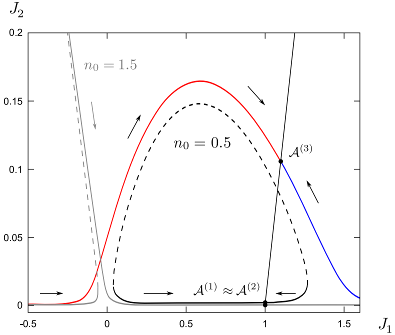
$$\boxed{m_{\parallel} = \frac{\gamma^3}{1 + 2\mu\Omega - dJ_1/dJ_2(k - p\Omega + 2k\Omega\mu)}}$$

- The regions of negative parallel mass are retained for $n \neq 1$:



Dissipative Forces

- Particle radiation and collisions limit feasibility of the negative mass effect
- Dissipation results in:
  - Drift of system states along the stationary surface
  - Attraction ($\gamma < 0$) or repulsion ($\gamma > 0$) from stationary surface $\boxed{\dot{\Lambda} = \gamma \Lambda}$
- For radiation friction:

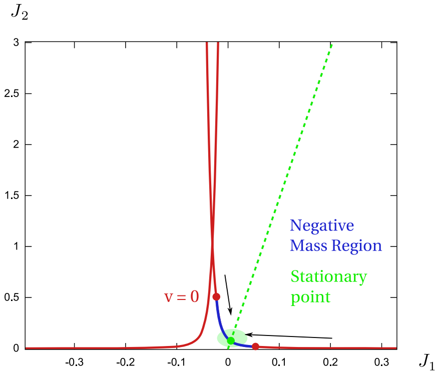


- Negative mass region is unstable (due to dissipations)
- Wave-particle Hamiltonian $\Rightarrow$ Wave energy evolution
- Wave energy dissipation for $m_\parallel > 0$
- Wave energy amplification for $m_\parallel < 0$ (supplied by particle kinetic energy)

◦ A. I. Zhmoginov, I. Y. Dodin, and N. J. Fisch, *Phys. Lett. A* **375**, 1236 (2011).

Effect of Collisions

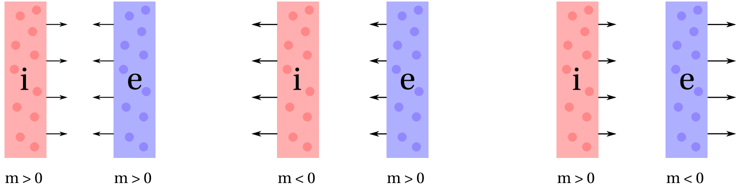
- Collisions $\Rightarrow$ Particle state drift and broadening of distribution function
- Collisions together with external forces can be used to bring particles to the **stabilized** negative mass region
- Stationary distribution function is localized near stationary curve if $m_{\text{negative}} \gg m_{\text{background}}$



- A. I. Zhmoginov, I. Y. Dodin, and N. J. Fisch, in preparation.

Negative Mass Instability

- Branch with $m_\parallel < 0 \Rightarrow$ Collective instability in plasma



- On the $m_\parallel < 0$ branch with $v \approx 0$:

$$\boxed{\omega_p^2 = \sum_s \frac{4\pi n_s q_s^2}{m_{s\,\parallel}} < 0}$$

- Condition for the instability:

$$\boxed{|n-1| \leq \varepsilon \left(8\widetilde{\mu}^{3/2}\right)^{-1}}$$

- A. I. Zhmoginov, I. Y. Dodin, and N. J. Fisch, *Phys. Rev. E* **81**, 036404 (2010).

Plasma Wave Manipulation

- Manipulating plasma wave by manipulating:
  - particles
  - laser wave
- Plasma wave group velocity:

$$\boxed{v_{\mathrm{gr}} = \frac{3k}{\omega_p} \sum_s \frac{\bar{\omega}_{p\,s}^2}{\omega_p^2} v_{\mathrm{th}\,s}^2}$$

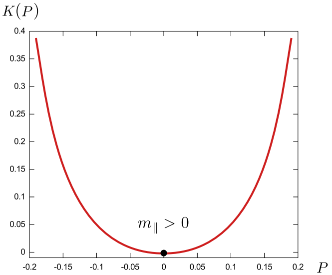
- Changing wave parameters one can keep $\langle v_s \rangle = 0$, but change v_{gr}
- v_{gr} can vanish in a system with two electron species (trapped and untrapped)

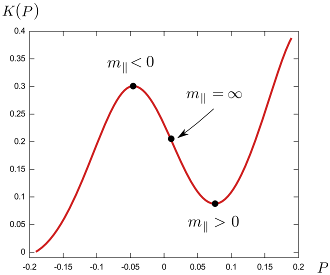
Kinetic Energy and the Negative Mass

- Drift particle Hamiltonian

$$\boxed{H(P,z) = K(P) + U(z)}$$

- Here $K(P)$ is a kinetic energy and $U(z)$ is a potential energy
- $K(P)$ profile corresponding to the negative mass case:

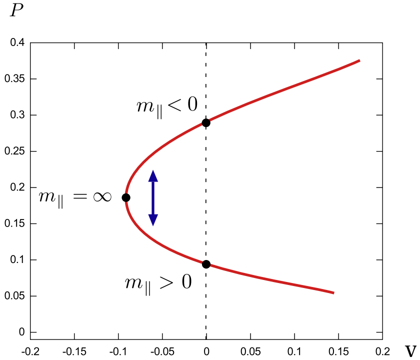




- Here $\boxed{v = \dot{z} = \partial K/\partial P}$ and $\boxed{m_{\|} \sim (\partial^2 K/\partial P^2)^{-1}}$

Frequency Doubling

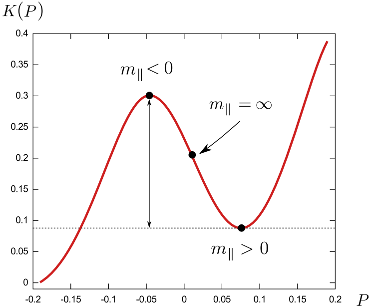
- $v(P)$ dependence:



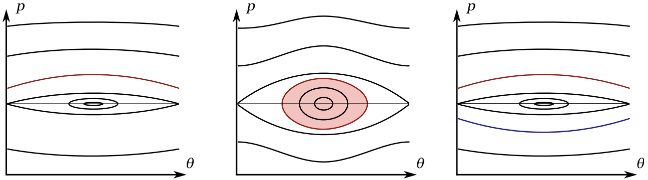
- $\omega_{\text{force}} = \omega_0 \Rightarrow \omega_{\text{particle}} = 2\omega_0$
- Particle radiates at $2\omega_0 \Rightarrow$ frequency doubling

WAVE-PARTICLE INTERACTION

- State with $v = 0$ and $m_{\|} < 0$ has larger energy:

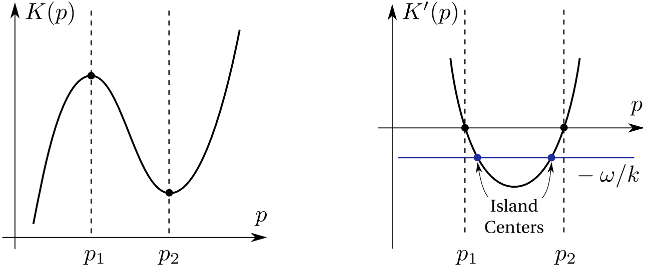


- Separatrix-crossing effect to extract this energy:

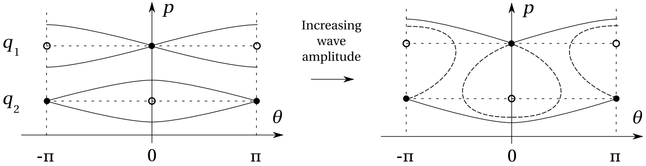


WAVE-PARTICLE INTERACTION

- Two island chains for $m_{\|} < 0$:



- Two phase-space configurations:



WAVE-PARTICLE INTERACTION

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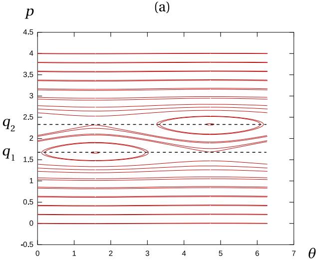
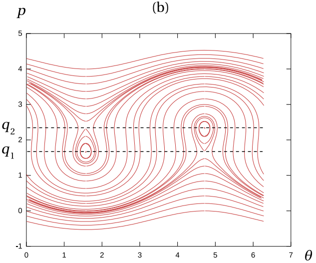
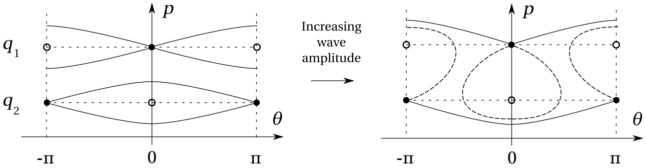


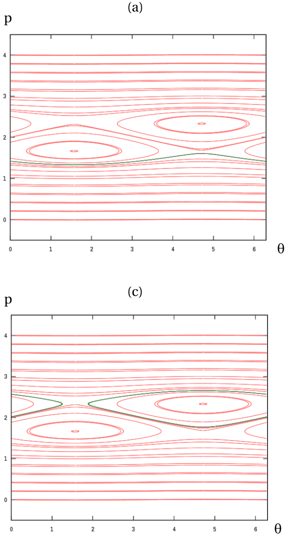


- Two phase-space configurations:



Wave-Particle Interaction

- Particle passes several separatrix-crossing events:



Wave-Particle Interaction

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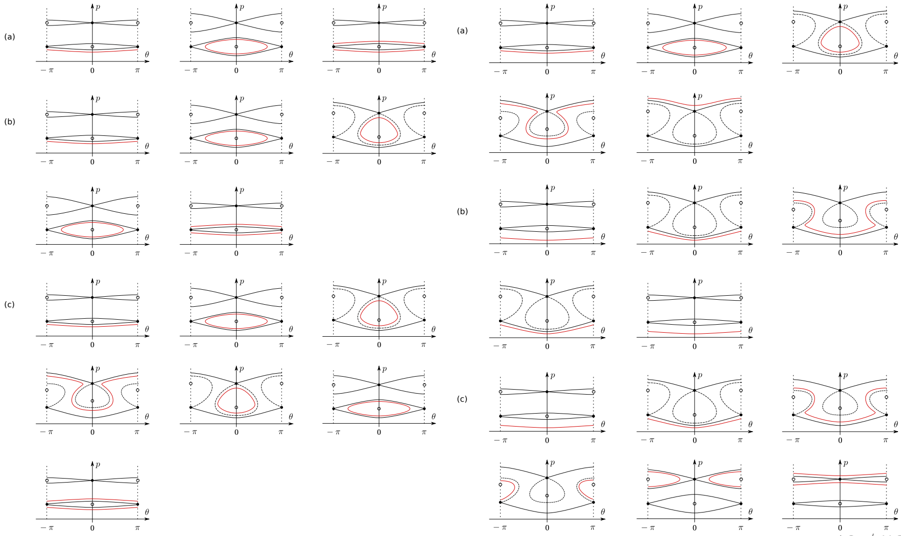
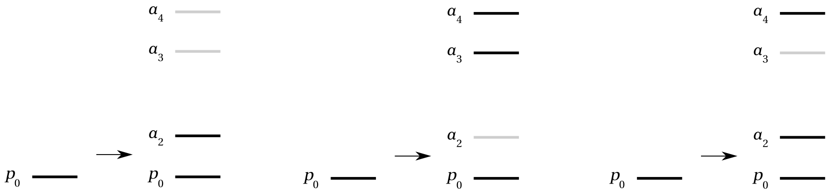


Wave-Particle Interaction

- Two or three final levels and Three types of transitions



- Some particles can be transfered from one state with $v = 0$ to another
- Energy difference $\rightarrow$ wave (amplifying it)
- Some particles can leave in the third state with $v \neq 0$
- Porbability of this is smaller when the trapping occurs at higher wave amplitudes

Main Results

Main Results

- General formalism for: effective parallel mass, dissipative and stochastic effects
- Negative mass effect is shown to be retained for $n \neq 1$
- The limitation on μ, above which states are unstable, is derived
- Radiation friction either accelerates particles ($p_{\|}$) or heats them (μ)
- Particles with $m_{\|} < 0$ are shown to be unstable due to radiation friction
- The radiation friction is shown to cause wave to amplify when $m_{\|} < 0$
- Collisions with ligher particles coupled with external forces are shown to bring particles to $m_{\|} < 0$ branch
- Negative particle mass is shown to give rise to many interesting phenomena:
    - negative mass plasma instability
    - plasma wave manipulation
    - frequency doubling
    - birth of two islands in wave-particle interaction problems

Alpha-Channeling Simulations

Numerical Models

- Particle motion equations:

$$m\frac{d\boldsymbol{v}}{dt} = \frac{q}{c}\left(\boldsymbol{v} \times \boldsymbol{B}\right) - q\nabla\varphi.$$

- Random-walk equation:

$$\boldsymbol{v}_{n+1} = \boldsymbol{v}_n + \boldsymbol{f} + \hat{\boldsymbol{d}}\,\boldsymbol{w},$$

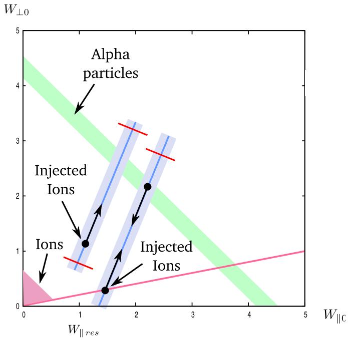
$$t_{n+1} = t_n + \Delta t(\boldsymbol{v}_n),$$

- Fokker-Planck equation:

$$\frac{\partial p}{\partial t} = -\sum_i \frac{\partial}{\partial v_i}\left(\frac{p f_i}{\Delta t}\right) + \frac{1}{2}\sum_{i,j}\frac{\partial^2}{\partial v_i\,\partial v_j}\left[\frac{p}{\Delta t}\left(\hat{\boldsymbol{d}}\hat{\boldsymbol{d}}^T\right)_{ij}\right].$$

Ion Heating

- Deuterium ions and α particles have the same q/m ratio. Fuel ions can be injected along the same path that is used to cool down α particles. Energy extracted from α particles is then used to heat ions.
- Ion loss can be avoided by limiting ion cooling (controlled by the radial extent of the rf region). Alternatively, ions can be injected along a different diffusion path.



Magnetic Field Profile

We assume that the magnetic field $\boldsymbol{B}$ in both devices is given by $B_\phi = 0$, $B_r = -r(dB_z/dz)/2$, and

$$B_z = B_{\min} + \frac{1}{2}(B_{\max} - B_{\min})\left[1 - \cos\left(\pi\left|2\eta z/L\right|^g\right)\right],$$

where g is an integer, $\eta \geq 1$ is a constant, $B_{\min}$ and $B_{\max}$ are the minimum and the maximum values of B_z correspondingly.

Plasma Dispersion Relation

We assume that:

- the linear density of the plasma does not depend on the axial position $n(z)|_{R=0} \approx n^0 B(z)/B_0$ on the axis, and
- that radial plasma temperature and density profiles are given by $n(\boldsymbol{r}, z) = n(z)|_{R=0} \exp(-R^2/a^2)$ and $T(\boldsymbol{r}) = T^0[\kappa + (1-\alpha)\exp(-R^2/a^2)]$, where $\alpha \leq 1$ is a constant, and a is a characteristic plasma radius.

Plasma Dispersion Relation

The dispersion relation $\mathcal{D}=0$ is modelled by the plasma kinetic dispersion relation reading $\mathcal{D}=\|\hat{\boldsymbol{\varepsilon}}-n^2\hat{\mathbf{1}}+\boldsymbol{nn}\|$, where $\hat{\boldsymbol{\varepsilon}}=\hat{\mathbf{1}}+\sum_s\hat{\boldsymbol{\chi}}_s$, $\hat{\boldsymbol{\chi}}_s=\omega_{ps}^2/\omega\cdot\sum_n e^{-\lambda}\hat{\boldsymbol{Y}}_n^s(\lambda)$, and tensor $\hat{\boldsymbol{Y}}_n^s(\lambda)$ is given by the following expression:

$$\hat{\boldsymbol{Y}}_n^s=\begin{pmatrix} \dfrac{n^2 I_n}{\lambda_s}A_n & -in\Delta I_n A_n & \dfrac{k_\perp}{\Omega_s}\dfrac{nI_n}{\lambda_s}B_n \\ in\Delta I_n A_n & QA_n & \dfrac{ik_\perp}{\Omega_s}\Delta I_n B_n \\ \dfrac{k_\perp}{\Omega_s}\dfrac{nI_n}{\lambda_s}B_n & -\dfrac{ik_\perp}{\Omega_s}\Delta I_n B_n & \dfrac{2(\omega-n\Omega_s)}{k_\parallel w_{s\perp}^2}I_n B_n \end{pmatrix}.$$

◇ T. H. Stix, *Waves in Plasmas* (Springer-Verlag, New York, 1992).

Plasma Dispersion Relation

Here ω_{ps}^2 is the plasma frequency for species s, $Q = \left(n^2 I_n \lambda_s^{-1} + 2\lambda_s \Delta I_n\right)$, $\Delta I_n = I_n(\lambda_s) - I_n'(\lambda_s)$, $A_n = (k_\| w_{s\,\|})^{-1} Z_0(\xi_n^s)$, $B_n = k_\|^{-1}[1 + \xi_n^s Z_0(\xi_n^s)]$, $\xi_n^s = (\omega - n\Omega_s)(k_\| w_{s\,\|})^{-1}$, $\lambda_s = k_\perp^2 \rho_s^2/2$, Z_0 is the real part of the plasma dispersion function, $w_{s\,\|}$ and $w_{s\,\perp}$ are parallel and perpendicular thermal particle velocities correspondingly, $\rho_s = w_{s\,\perp}/\Omega_s$, and Ω_s is the gyrofrequency.

Ray-Tracing Equations

- The corresponding Hamiltonian equations are

$$\dot{R} = \alpha \frac{\partial \mathcal{D}}{\partial k_n}, \qquad \dot{\varkappa} = \beta \frac{\partial \mathcal{D}}{\partial k_\parallel},$$

$$\dot{K}_R = -\frac{\partial \mathcal{D}}{\partial R} - K_R \frac{\partial \alpha}{\partial R} \frac{\partial \mathcal{D}}{\partial k_n} - K_\varkappa \frac{\partial \beta}{\partial R} \frac{\partial \mathcal{D}}{\partial k_\parallel},$$

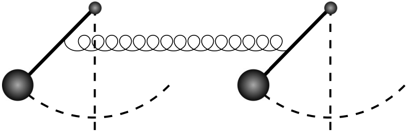
$$\dot{K}_\varkappa = -\frac{\partial \mathcal{D}}{\partial \varkappa} - K_R \frac{\partial \alpha}{\partial \varkappa} \frac{\partial \mathcal{D}}{\partial k_n} - K_\varkappa \frac{\partial \beta}{\partial \varkappa} \frac{\partial \mathcal{D}}{\partial k_\parallel}.$$

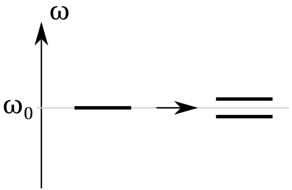
- For mirror machines with $R \ll L$:

$$\alpha \approx \sqrt{B/B_0}, \qquad \beta = 1 + O(R^2).$$

Mode Coupling

- The extracted energy can also be coupled to particles in the mirror plug.
- Consider two harmonic oscillators with the same frequency ω:
    - If the oscillators are uncoupled, there is no energy transfer between them.
    - If an arbitrarily small coupling is introduced, the energy can be transfered between the oscillators.
      The energy levels split and the initial state with all energy in one of the oscillators corresponds to the superposition state.





Mode Coupling

- The extracted energy can also be coupled to particles in the mirror plug.
- Consider two harmonic oscillators with the same frequency ω:
  - If the oscillators are uncoupled, there is no energy transfer between them.
  - If an arbitrarily small coupling is introduced, the energy can be transfered between the oscillators.
    The energy levels split and the initial state with all energy in one of the oscillators corresponds to the superposition state.